Introduction to Image Processing with SciPy and NumPy

Anil C R
cranil@ee.iisc.ernet.in

Department of Electrical Engineering
Indian Institute of Science

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Outline

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What are Images?

Continious domain, Continious range

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}(\mathbb{R}^3)$$

Discrete domain, Continious range

$$f : \mathbb{Z}^2 \rightarrow \mathbb{R}(\mathbb{R}^3)$$

Discrete domain, Discrete range

$$f : \mathbb{Z}^2 \rightarrow \mathbb{Z}(\mathbb{R}^3)$$

Finite domain, Continious range

$$f : \mathbb{Z}_m \times \mathbb{Z}_n \rightarrow \mathbb{R}(\mathbb{R}^3)$$

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Using Matrices to Represent Images

- f as an element of $\mathbb{R}^{m \times n}(\mathbb{R}^{m \times n \times k})$
- $\Rightarrow$ Linear Algebra
- $\Rightarrow$ LAPACK, BLAS, etc
- $\Rightarrow$ FORTRAN, C, etc
- $\Rightarrow$ Super Hard
- $\Rightarrow$ MATLAB
- $\Rightarrow$ Super Expensive
- $\Rightarrow$ SciPy + NumPy, GNU Octave, Scilab, etc
- PyCon 2010
- $\Rightarrow$ SciPy + NumPy

Outline

NumPy

- Numerical Processing
- Started off as *numecric* written in 1995 by Jim Huguni et al.
- Numeric was slow for large arrays and was rewritten for large arrays as *Numarray*
- Travis Oliphant, in 2005 merged them both into *NumPy*

SciPy

- Libraries for scientific computing
- Linear Algebra
- Statistics
- Signal and Image processing
- Optimization
- ODE Solvers

Outline

1. Introduction
   - Image Processing
   - What are SciPy and NumPy?
2. Some Theory
   - Filters
   - The Fourier Transform
3. Doing the Stuff in Python
4. Demo(s)

Filters

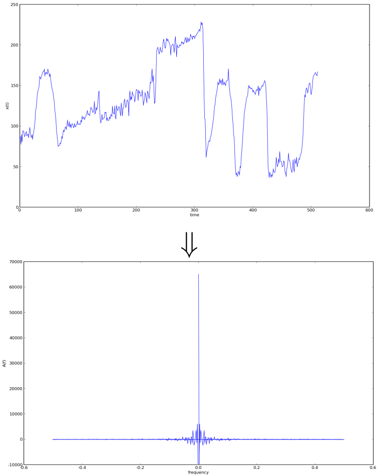
- Keep what you want and throw away the rest
- Studying filters is the most important part in Image Processing
- Classified into *linear* and *non-linear* filters

Linear Filters

- Given images $f_1, f_2, \ldots, f_n$ a filter H is called linear if $H(\alpha_1 f_1 + \alpha_2 f_2 + \cdots + \alpha_n f_n) =$
$$\alpha_1 H(f_1) + \alpha_2 H(f_2) + \cdots + \alpha_n H(f_n)$$
- Linearity can be useful in fast computation

Outline

Time and Frequency Domains



Fourier Transform

Continious FT

$$F(\omega_x x, \omega_y y) = \int\int_{\mathbb{R}^2} f(x, y) exp(-i\omega_x x - i\omega_y y) dxdy$$

Discrete FT

$$F(\omega_x x, \omega_y y) = \sum_0^M \sum_0^N f(x, y) exp(-i\{\frac{2\pi\omega_x}{M} x - \frac{2\pi\omega_y}{N} y\})$$

Notation: F is the FT of f, also $F = \mathcal{F}\{f\}$

Convolution

Continious

$$(f \circledast g)(x, y) = \iint_{\mathbb{R}^2} f(x', y')g(x - x', y - y')dxdy$$

Discrete

$$(f \circledast g)(x, y) = \sum\sum_{\mathbb{Z}^2} f(x, y)g(x - x', y - y')$$

Theorem

(a) $\mathcal{F}\{f \circledast g\} = FG$

(b) $\mathcal{F}\{fg\} = F \circledast G$

Any linear filter can be written as a convolution.

Fast Fourier Transform (FFT)

- Computing the Discrete Fourier Transform takes $O(n^2m^2)$ for an $m \times n$ image
- FFT Computes the same in $O(n \log nm \log m)$

Interactive Python

- Install NumPy
- Install SciPy
- Install Matplotlib
- Install IPython

Running IPython

```
$ ipython -pylab
```

Fast Fourier Transform (FFT)

FFT in NumPy

```
In[1]: from scipy import lena
In[2]: f = lena()
In[3]: from numpy.fft import fft2 #
unnecessary if you invoke ipython with -pylab
In[4]: F = fft2(f)
In[5]: imshow(real(F))
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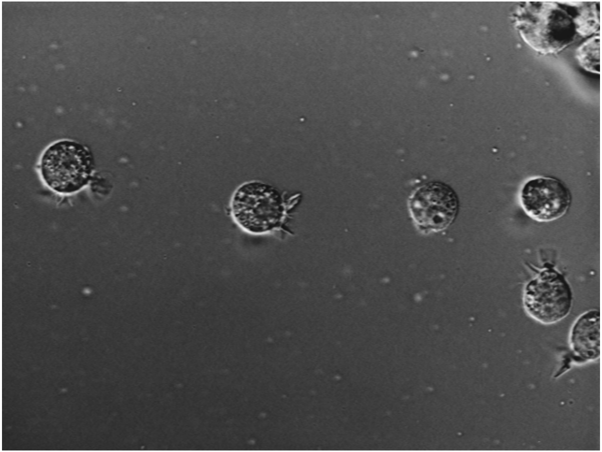
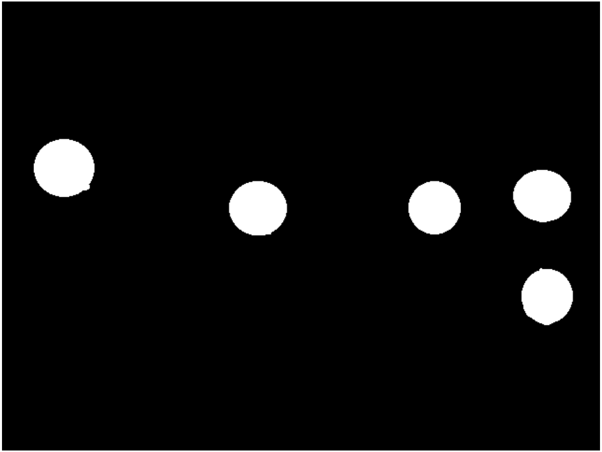
Demo: Cells



Input Image

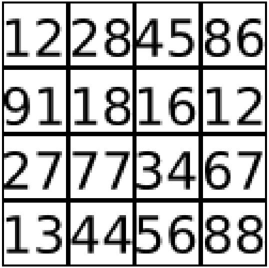
Demo: Cells



Output

Demo: Cells
Procedure

Consider the image:



Demo: Cells

Procedure Cont'd

Find the variance of the neighborhood of each pixel, store them as a 2D array

12	28	45	86
91	18	16	12
27	77	34	67
13	44	56	88

Demo: Cells

Procedure Cont'd

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Demo: Cells

Procedure Cont'd

Variance map (V)

$V > \mathbb{E}\{V\}$

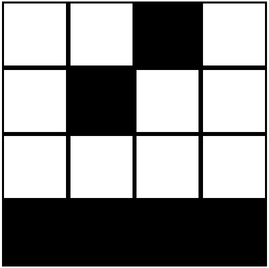
996	744	654	875
920	686	689	700
870	670	695	750
571	426	344	380

Demo: Cells

Procedure Cont'd

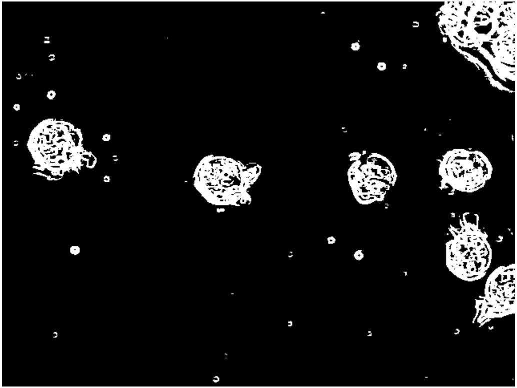
Variance map (V)

$V > \mathbb{E}\{V\}$



Demo: Cells
Procedure Cont'd

Algorithm on the cell image:



Q and A